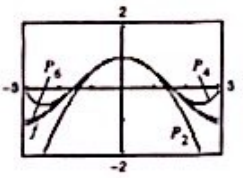
11. $f(x) = \cos x$

$P_2(x) = 1 - \frac{1}{2}x^2$

$P_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4$

$P_6(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6$

(a)



(b) $f'(x) = -\sin x \qquad P_2'(x) = -x$

$f''(x) = -\cos x \qquad P_2''(x) = -1$

$f''(0) = P_2''(0) = -1$

$f'''(x) = \sin x \qquad P_4'''(x) = x$

$f^{(4)}(x) = \cos x \qquad P_4^{(4)}(x) = 1$

$f^{(4)}(0) = 1 = P_4^{(4)}(0)$

$f^{(5)}(x) = -\sin x \qquad P_6^{(5)}(x) = -x$

$f^{(6)}(x) = -\cos x \qquad P^{(6)}(x) = -1$

$f^{(6)}(0) = -1 = P_6^{(6)}(0)$

(c) In general, $f^{(n)}(0) = P_n^{(n)}(0)$ for all n.

12. $f(x) = x^2e^x, f(0) = 0$

(a) $f'(x) = (x^2 + 2x)e^x \qquad f'(0) = 0$

$f''(x) = (x^2 + 4x + 2)e^x \qquad f''(0) = 2$

$f'''(x) = (x^2 + 6x + 6)e^x \qquad f'''(0) = 6$

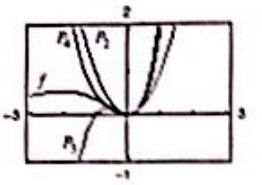
$f^{(4)}(x) = (x^2 + 8x + 12)e^x \qquad f^{(4)}(0) = 12$

$P_2(x) = \dfrac{2x^2}{2!} = x^2$

$P_3(x) = x^2 + \dfrac{6x^3}{3!} = x^2 + x^3$

$P_4(x) = x^2 + x^3 + \dfrac{12x^4}{4!} = x^2 + x^3 + \dfrac{x^4}{2}$

(b)



(c) $f''(0) = 2 = P_2''(0)$

$f'''(0) = 6 = P_3'''(0)$

$f^{(4)}(0) = 12 = P_4^{(4)}(0)$

(d) $f^{(n)}(0) = P_n^{(n)}(0)$

13. $f(x) = e^{-x} \qquad f(0) = 1$

$f'(x) = -e^{-x} \qquad f'(0) = -1$

$f''(x) = e^{-x} \qquad f''(0) = 1$

$f'''(x) = -e^{-x} \qquad f'''(0) = -1$

$P_3(x) = f(0) + f'(0)x + \dfrac{f''(0)}{2!}x^2 + \dfrac{f'''(0)}{3!}x^3$

$= 1 - x + \dfrac{x^2}{2} - \dfrac{x^3}{6}$

14. $f(x) = e^{-x} \qquad f(0) = 1$

$f'(x) = -e^{-x} \qquad f'(0) = -1$

$f''(x) = e^{-x} \qquad f''(0) = 1$

$f'''(x) = -e^{-x} \qquad f'''(0) = -1$

$f^{(4)}(x) = e^{-x} \qquad f^{(4)}(0) = 1$

$f^{(5)}(x) = -e^{-x} \qquad f^{(5)}(0) = -1$

$P_5(x) = f(0) + f'(0)x + \dfrac{f''(0)}{2!}x^2 + \dfrac{f'''(0)}{3!}x^3 + \dfrac{f^{(4)}(0)}{4!}x^4$

$+ \dfrac{f^{(5)}(0)}{5!}x^5 = 1 - x + \dfrac{x^2}{2} - \dfrac{x^3}{6} + \dfrac{x^4}{24} - \dfrac{x^5}{120}$

15. $f(x) = e^{2x} \qquad f(0) = 1$

$f'(x) = 2e^{2x} \qquad f'(0) = 2$

$f''(x) = 4e^{2x} \qquad f''(0) = 4$

$f'''(x) = 8e^{2x} \qquad f'''(0) = 8$

$f^{(4)}(x) = 16^{2x} \qquad f^{(4)}(0) = 16$

$P_4(x) = 1 + 2x + \dfrac{4}{2!}x^2 + \dfrac{8}{3!}x^3 + \dfrac{16}{4!}x^4$

$= 1 + 2x + 2x^2 + \dfrac{4}{3}x^3 + \dfrac{2}{3}x^4$

16. $f(x) = e^{3x} \qquad f(0) = 1$

$f'(x) = 3e^{3x} \qquad f'(0) = 3$

$f''(x) = 9e^{3x} \qquad f''(0) = 9$

$f'''(x) = 27e^{3x} \qquad f'''(0) = 27$

$f^{(4)}(x) = 81e^{3x} \qquad f^{(4)}(0) = 81$

$P_4(x) = 1 + 3x + \dfrac{9}{2!}x^2 + \dfrac{27}{3!}x^3 + \dfrac{81}{4!}x^4$

$= 1 + 3x + \dfrac{9}{2}x^2 + \dfrac{9}{2}x^3 + \dfrac{27}{8}x^4$

17. $f(x) = \sin x$ $\quad f(0) = 0$

$f'(x) = \cos x$ $\quad f'(0) = 1$

$f''(x) = -\sin x$ $\quad f''(0) = 0$

$f'''(x) = -\cos x$ $\quad f'''(0) = -1$

$f^{(4)}(x) = \sin x$ $\quad f^{(4)}(0) = 0$

$f^{(5)}(x) = \cos x$ $\quad f^{(5)}(0) = 1$

$$P_5(x) = 0 + (1)x + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{0}{4!}x^4 + \frac{1}{5!}x^5$$

$$= x - \frac{1}{6}x^3 + \frac{1}{120}x^5$$

18. $f(x) = \sin \pi x$ $\quad f(0) = 0$

$f'(x) = \pi \cos \pi x$ $\quad f'(0) = \pi$

$f''(x) = -\pi^2 \sin \pi x$ $\quad f''(0) = 0$

$f'''(x) = -\pi^3 \cos \pi x$ $\quad f'''(0) = -\pi^3$

$$P_3(x) = 0 + \pi x + \frac{0}{2!}x^2 + \frac{-\pi^3}{3!}x^3 = \pi x - \frac{\pi^3}{6}x^3$$

19. $f(x) = xe^x$ $\quad f(0) = 0$

$f'(x) = xe^x + e^x$ $\quad f'(0) = 1$

$f''(x) = xe^x + 2e^x$ $\quad f''(0) = 2$

$f'''(x) = xe^x + 3e^x$ $\quad f'''(0) = 3$

$f^{(4)}(x) = xe^x + 4e^x$ $\quad f^{(4)}(0) = 4$

$$P_4(x) = 0 + x + \frac{2}{2!}x^2 + \frac{3}{3!}x^3 + \frac{4}{4!}x^4$$

$$= x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$$

20. $f(x) = x^2e^{-x}$ $\quad f(0) = 0$

$f'(x) = 2xe^{-x} - x^2e^{-x}$ $\quad f'(0) = 0$

$f''(x) = 2e^{-x} - 4xe^{-x} + x^2e^{-x}$ $\quad f''(0) = 2$

$f'''(x) = -6e^{-x} + 6xe^{-x} - x^2e^{-x}$ $\quad f'''(0) = -6$

$f^{(4)}(x) = 12e^{-x} - 8xe^{-x} + x^2e^{-x}$ $\quad f^{(4)}(0) = 12$

$$P_4(x) = 0 + 0x + \frac{2}{2!}x^2 + \frac{-6}{3!}x^3 + \frac{12}{4!}x^4$$

$$= x^2 - x^3 + \frac{1}{2}x^4$$

21. $f(x) = \dfrac{1}{x+1}$ $\quad f(0) = 1$

$f'(x) = -\dfrac{1}{(x+1)^2}$ $\quad f'(0) = -1$

$f''(x) = \dfrac{2}{(x+1)^3}$ $\quad f''(0) = 2$

$f'''(x) = \dfrac{-6}{(x+1)^4}$ $\quad f'''(0) = -6$

$f^{(4)}(x) = \dfrac{24}{(x+1)^5}$ $\quad f^{(4)}(0) = 24$

$$P_4(x) = 1 - x + \frac{2}{2!}x^2 + \frac{-6}{3!}x^3 + \frac{24}{4!}x^4$$

$$= 1 - x + x^2 - x^3 + x^4$$

22. $f(x) = \dfrac{x}{x+1} = \dfrac{x+1-1}{x+1}$ $\quad f(0) = 0$

$= 1 - (x+1)^{-1}$

$f'(x) = (x+1)^{-2}$ $\quad f'(0) = 1$

$f''(x) = -2(x+1)^{-3}$ $\quad f''(0) = -2$

$f'''(x) = 6(x+1)^{-4}$ $\quad f'''(0) = 6$

$f^{(4)}(x) = -24(x+1)^{-5}$ $\quad f^{(4)}(0) = -24$

$$P_4(x) = 0 + 1(x) - \frac{2}{2}x^2 + \frac{6}{6}x^3 - \frac{24}{24}x^4$$

$$= x - x^2 + x^3 - x^4$$

23. $f(x) = \sec x$ $\quad f(0) = 1$

$f'(x) = \sec x \tan x$ $\quad f'(0) = 0$

$f''(x) = \sec^3 x + \sec x \tan^2 x$ $\quad f''(0) = 1$

$$P_2(x) = 1 + 0x + \frac{1}{2!}x^2 = 1 + \frac{1}{2}x^2$$

24. $f(x) = \tan x$ $\quad f(0) = 0$

$f'(x) = \sec^2 x$ $\quad f'(0) = 1$

$f''(x) = 2 \sec^2 x \tan x$ $\quad f''(0) = 0$

$f'''(x) = 4 \sec^2 x \tan^2 x + 2 \sec^4 x$ $\quad f'''(0) = 2$

$$P_3(x) = 0 + 1(x) + 0 + \frac{2}{6}x^3 = x + \frac{1}{3}x^3$$

25. $f(x) = \frac{1}{x}$ $f(1) = 1$

$f'(x) = -\frac{1}{x^2}$ $f'(1) = -1$

$f''(x) = \frac{2}{x^3}$ $f''(1) = 2$

$f'''(x) = -\frac{6}{x^4}$ $f'''(1) = -6$

$f^{(4)}(x) = \frac{24}{x^5}$ $f^{(4)}(1) = 24$

$$P_4(x) = 1 - (x-1) + \frac{2}{2!}(x-1)^2 + \frac{-6}{3!}(x-1)^3 + \frac{24}{4!}(x-1)^4$$

$$= 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4$$

26. $f(x) = 2x^{-2}$ $f(2) = \frac{1}{2}$

$f'(x) = -4x^{-3}$ $f'(2) = -\frac{1}{2}$

$f''(x) = 12x^{-4}$ $f''(2) = \frac{3}{4}$

$f'''(x) = -48x^{-5}$ $f'''(x) = -\frac{3}{2}$

$f^{(4)}(x) = 240x^{-6}$ $f^{(4)}(x) = \frac{15}{4}$

$$P_4(x) = \frac{1}{2} - \frac{1}{2}(x-2) + \frac{3}{8}(x-2)^2 - \frac{1}{4}(x-2)^3 + \frac{5}{32}(x-2)^4$$

27. $f(x) = \sqrt{x}$ $f(1) = 1$

$f'(x) = \frac{1}{2\sqrt{x}}$ $f'(1) = \frac{1}{2}$

$f''(x) = -\frac{1}{4x\sqrt{x}}$ $f''(1) = -\frac{1}{4}$

$f'''(x) = \frac{3}{8x^2\sqrt{x}}$ $f'''(1) = \frac{3}{8}$

$f^{(4)}(x) = -\frac{15}{16x^3\sqrt{x}}$ $f^{(4)}(1) = -\frac{15}{16}$

$$P_4(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4$$

28. $f(x) = x^{1/3}$ $f(8) = 2$

$f'(x) = \frac{1}{3}x^{-2/3}$ $f'(8) = \frac{1}{12}$

$f''(x) = -\frac{2}{9}x^{-5/3}$ $f''(8) = -\frac{1}{144}$

$f'''(x) = \frac{10}{27}x^{-8/3}$ $f'''(8) = \frac{10}{27} \cdot \frac{1}{2^8} = \frac{5}{3456}$

$$P_3(x) = 2 + \frac{1}{12}(x-8) - \frac{1}{288}(x-8)^2 + \frac{5}{20{,}736}(x-8)^3$$

29. $f(x) = \ln x$ $f(1) = 0$

$f'(x) = \frac{1}{x}$ $f'(1) = 1$

$f''(x) = -\frac{1}{x^2}$ $f''(1) = -1$

$f'''(x) = \frac{2}{x^3}$ $f'''(1) = 2$

$f^{(4)}(x) = -\frac{6}{x^4}$ $f^{(4)}(1) = -6$

$$P_4(x) = 0 + (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$$

30. $f(x) = x^2 \cos x$ $f(\pi) = -\pi^2$

$f'(x) = \cos x - x^2 \sin x$ $f'(\pi) = -2\pi$

$f''(x) = 2\cos x - 4x \sin x - x^2 \cos x$ $f''(\pi) = -2 + \pi^2$

$$P_2(x) = -\pi^2 - 2\pi(x-\pi) + \frac{(\pi^2 - 2)}{2}(x-\pi)^2$$